Let $2x=x'$, then

$$\int_0^{\frac{1}{2}\pi}\log\sin 2x\,dx=\tfrac{1}{2}\int_0^{\pi}\log\sin x'\,dx'=\int_0^{\frac{1}{2}\pi}\log\sin x\,dx.$$

$$\therefore 2u=u-\frac{\pi}{2}\log 2,\quad u=-\frac{\pi}{2}\log 2=\frac{\pi}{2}\log\tfrac{1}{2}.$$

Also solved by M. E. Graber, G. W. Greenwood, L. E. Newcomb, and G. B. M. Zerr.

206. Proposed by DR. O. E. GLENN, Drury College.

Evaluate $\int_0^1(1-z^n)^m\frac{\partial}{\partial z}\log(1-z^nx^n)dz$, assuming $-1<x^n<+1$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$u=-\int_0^1\frac{nx^nz^{n-1}(1-z^n)^m}{1-x^nz^n}dz$. Let $1-z^n=y$, then we have

$$u=-\int_0^1\frac{x^ny^m}{1-x^n+x^ny}dy=-\int_0^1\frac{y^m}{y+a}dy,\text{ where }\frac{1-x^n}{x^n}=a.$$

$$\therefore u=-\int_0^1\left(y^{m-1}-ay^{m-2}+a^2y^{m-3}.....(-1)^{m-1}a^{m-1}+\frac{(-1)^ma^m}{y+a}\right)dy$$

$$=-\left.\frac{y^m}{m}-\frac{ay^{m-1}}{m-1}+.....+(-1)^{m-1}a^{m-1}y+(-1)^ma^m\log(y+a)\right]_0^1$$

$$=-\left[\frac{1}{m}-\frac{a}{m-1}+.....+(-1)^{m-1}a^{m-1}+(-1)^ma^m\log\left(\frac{1+a}{a}\right)\right]$$

$$=-\left[\frac{1}{m}-\frac{(1-x^n)}{x^n(m-1)}+\frac{(1-x^n)^2}{x^{2n}(m-2)}+.....\frac{(-1)^{m-1}(1-x^n)^{m-1}}{x^{(m-1)n}}\right.$$

$$\left.+(-1)^{m+1}\frac{(1-x^n)^m}{x^{mn}}\log(1-x^n)\right].$$

DIOPHANTINE ANALYSIS.

128. Proposed by F. P. MATZ, Ph. D., Sc. D., Reading, Pa.

Required the highest powers of 2, 3, 5, 7, contained in (1000)!

I. Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

$$(1000)!=2^{500}(500)!(1.3.5\,........\,999)$$
$$(500)!=2^{250}(250)!(1.3.5\,.........\,499)$$

....

Proceeding thus we find the powers required are

$$2^{994},\quad 3^{498},\quad 5^{249},\quad 7^{164}.$$

II. Solution by DR. O. E. GLENN, Drury College.

The theorem covering the general problem is due to Gauss,* and is the following: If p is any prime less than or equal to m, then the highest power of p dividing $m!$ is

$$p^{[m/p]+[m/p^2]+[m/p^3]+\ldots\ldots}=p^{\sum_{i=1}^{\infty}[m/p^i]}$$

where $[s/t]$ stands for the greatest integer in s/t. Applying this we have $2^{500+250+125+62+31+15+7+3+1}=2^{994}$, and similarly for the others.

Also solved by A. H. Holmes, and G. B. M. Zerr.

GEOMETRY.

263. Proposed by FREDERICK R. HONEY, Trinity College, Hartford, Conn.

Construct a sphere whose surface shall intersect the surface of any four given spheres in great circles.

Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Let P be the center of a circle intersecting two circles, centers C, C', in the extremities of diameters AB, $A'B'$, respectively. Draw through P a perpendicular to CC', intersecting it in D. Then, if r be the radius of the intersecting circle, we have

$$r^2=PC^2+CA^2=PC'^2+C'A'^2.$$

$$\therefore PD^2+DC^2+CA^2=PD^2+DC'^2+C'A'^2, \text{ and } DC^2-DC'^2=C'A'^2-CA^2.$$

Hence D is a fixed point, and the locus of P is consequently a fixed line. By rotating the figure about CC' we find that the locus of the center of a sphere intersecting two given spheres in great circles is a certain plane.

Constructing these planes for three pairs of the given spheres, each sphere being involved, we get a common point as the center of the required sphere, assuming that the centers of the given spheres are not coplanar.

264. Proposed by B. F. FINKEL, A. M., Drury College, Springfield, Mo.

Let l and m be two straight lines intersecting in A. With A as center and any radius r describe a circle intersecting l and m in E, M and G, Q, respectively; and the bisector of the opposite angles formed by l and m in F and K. With I, the middle point of EA, as center, and radius, r, describe an arc intersecting the bisector of the opposite angles formed by l and m in O. With O as center, and radius $OA+r$ describe circle $FHCDBJF$; F and D the points of intersection of this circle with the bisector of opposite angle; H, B the intersections on l, and J, C on m. What is the ratio of arc HFJ to arc BD?

Disquisitiones Arithmeticae.